NAVAL POSTGRADUATE SCHOOL
Monterey, California

FLOW INTO A TRANSONIC COMPRESSOR ROTOR

Part 1 - Analysis

R. P. Shreeve

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An analytical representation is given for the axisymmetric 2-dimensional flow field at the face of a transonic rotor. Streamlines computed for irrotatational inviscid flow are closely approximated using a curve-fitting technique, which is described, and boundary layer blockage is then introduced. With the inlet flow so represented, the conditions relative to the rotor blading of the TRANSX compressor are calculated as a function of the radius and graphed as a function of flow rate and rotor speed.

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The purpose of this report is to both document calculations carried out for the TRANSX compressor and to record useful analytical techniques and short programs developed using the Hewlett-Packard Model 9830A Calculator.

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1. INTRODUCTION

An essential feature of transonic "axial" compressor stages is the contraction of the flow passage approaching and through the blading. This is necessary since the stage pressure ratio is significantly greater than unity. In order to design blading, or to calculate inlet conditions during operation of a compressor, the radial distributions of velocity and flow angle are required in a radial plane immediately ahead of the blading. Axisymmetry is assumed.

The purpose of this report is to document calculations performed for the "TRANSX" compressor in the Turbopropulsion Laboratories at the Naval Postgraduate School. The various short programs developed in the process can be adapted easily to other geometries and to different applications. Extensive use has been made of the Hewlett-Packard Model 9830A Calculator. This type of programmable calculator allows performance and design calculations to be programmed very quickly by an individual investigator or engineer. A collection of these mini-programs will become the tools for a complete design analysis, and it is therefore advisable to document the approach as it evolves. The programs themselves are not listed here. It is the author's opinion that the programming is simple and best carried out by the user. However, the programs reported are identified by log number and can be listed or taped if requested.

The style of the analysis results from the repeated use of the "total flow function" which is reintroduced in Appendix A. In the analysis of internal compressible flows, particularly if they are adiabatic, the flow function provides the means of replacing density in the equations with a well-understood function of the velocity. With a technique for obtaining velocity from the flow function worked out, as described in Appendix A, the treatment of adiabatic internal flows as being compressible is little more elaborate than the treatment as incompressible, provided that a programmable calculator is available. Furthermore, the extreme precision of such calculators allows the use of the compressible flow formulation to very small velocities.

In Section 2 of this report, results are given for a calculation of inviscid, irrotational but compressible flow streamlines at the rotor face of the TRANSX compressor. Program IPP used in these calculations was written for the IBM360 computer. When reprogrammed for the HP9830A it will be reported separately. In Section 2, the computed distributions of velocity and flow angle are well represented by a particular curve fit described in Appendix B. The curve fitting procedure provides the exact value at the outer wall, control of the slope at the wall and an exact fit through three further points. The technique is useful when data is known accurately at only a few points. In Section 3 the effect of a boundary layer on the outer wall is represented by a blockage factor and related to the boundary layer displacement thickness. In Appendix C the blockage factor for the TRANSX compressor is evaluated from preliminary test results.

The analytical representation of the compressible flow in radial planes obtained in Sections 2 and 3 allows conditions radially along and relative to the rotor blading to be related to the total flow rate and compressor speed. In Section 4 the flow rate is related to the flow angle at the rotor tip for a given speed. The results graphed by the HP9830A in Figure 4 are used to show where the compressor is operating in relation to design incidence conditions. In Section 5, the distribution of the relative Mach number is calculated for a given flow angle at the tip and a given speed. The results graphed in Figure 5 are needed to guide the interpretation of flow field measurements as the compressor speed is increased.

2. INVISCID, IRROTATIONAL COMPRESSIBLE FLOW STREAMLINES

Compressible flow profiles were calculated using an IBM 360 computer program (IPP)[1] which assumes that stream surfaces are axisymmetric and locally parabolic in a radial plane. The position and slope of the streamlines given by Program IPP agreed closely with results of a flow net construction for incompressible flow. [The small variation from hub to tip in the velocity (0.25%) and therefore also in the density accounts for the agreement]. The calculation was done for the design flow rate corresponding to an axial Mach number at the tip of 0.646. It was therefore assumed that the pattern of streamlines was independent of the flow rate.

Referring to Figure 1, continuity at station 1 gives

$$\dot{w} = \int_{r_i}^{r_o} \rho V \cos\lambda \; 2\pi r dr \qquad (1)$$

which can be written as

$$\bar{w} = \dot{w}/(\rho_t V_t \pi r_o{}^2) = \int_{r_i}^{r_o} [\rho V/\rho_t V_t] \cdot \cos\lambda \, 2 \cdot (r/r_o) . d\,(r/r_o) \qquad (2)$$

where subscript t denotes "stagnation" values.

Using the definition of the "total flow function", Φ, given in Appendix A, and writing $R = r/r_o$, Eq. (2) becomes

$$\bar{w} = \int_{R_i}^{1} \Phi \cos\lambda \, d(R^2) \tag{2a}$$

which can be written as

$$\bar{w} = \Phi_o \cos\lambda_o \int_{R_i}^{1} (\Phi/\Phi_o)(\cos\lambda/\cos\lambda_o) \, d(R^2) \tag{3}$$

In the TRANSX compressor $\lambda_o = 0$, so that from Eq. (3),

$$\Phi_o = \bar{w}/\xi \tag{4}$$

where

$$\xi = \int_{R_i}^{1} (\Phi/\Phi_o) \cos\lambda \, d(R^2) \tag{4a}$$

and ξ is a function only of the inlet geometry.

Using the data from Program IPP, the distribution of the total flow function could be represented analytical by

$$(1 - R^2) = a_1\left(1 - \frac{\Phi}{\Phi_o}\right)^{n_1} + m_1\left(1 - \frac{\Phi}{\Phi_o}\right) \tag{5}$$

where the values

$$\left.\begin{aligned} a_1 &= 1.79864 \\ n_1 &= 0.38582 \\ m_1 &= 22.3048 \end{aligned}\right\} \tag{5a}$$

were found (Appendix B) to fit the data very closely. This is seen in Figure 2. Similarly the flow inclination was represented analytically by

$$\lambda = a_2(1 - R^2)^{n_2} + m_2(1 - R^2) \tag{6}$$

and the values

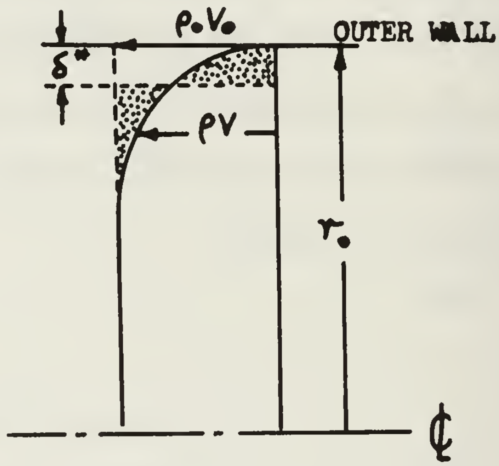
$$
\begin{aligned}
a_2 &= 0.9892665 \\
n_2 &= 3.8167895 \\
m_2 &= 0.4714926
\end{aligned}
\tag{6a}
$$

were found to fit the data very closely, and this is shown in Figure 3. The curve fitting method is described fully in Appendix B.

Using Eq. (5) and Eq. (6), the value of ξ can be obtained by integration of Eq. (4a). Since Eq. (5) cannot be solved explicitly for Φ/Φ_0 as a function of R, ξ was evaluated for the TRANSX compressor by graphically integrating the data from Program IPP. A value of ξ = 0.708355 was obtained.

3. BOUNDARY LAYER BLOCKAGE

Preliminary measurements of impact pressure ahead of the rotor have shown that a "boundary layer" exists on the outer wall extending approximately 0.25" into the flow. At smaller radii the stagnation pressure is nearly constant. In order to correct the inviscid streamline calculation for this deficit a blockage factor can be introduced which can be evaluated from measurements, or assumed a priori. If δ^* is the displacement thickness and $\delta^*/r_o \ll 1$,



$\dot{w} = \dot{w}_m + \rho_o V_o 2\pi r_o \delta^*$ where $\dot{w}_m$ is the measured flow rate and $\dot{w}$ is the flow rate for inviscid flow. Then, using the definition in Eq. (2),

$$\overline{W} = \overline{W}_m + \Phi_o 2\frac{\delta^*}{r_o}$$

Writing k_{B_1} for the area blockage factor usually defined as

$$k_B = \frac{\dot{w}_m}{\dot{w}} \tag{7}$$

then

$$\overline{W} = k_{B_1}\overline{W} + \Phi_o \cdot \frac{2\delta^*}{r_o}$$

or,

$$k_{B_1} = 1 - \frac{\Phi_o}{\overline{W}} \cdot \frac{2\delta^*}{r_o}$$

and using Eq. (4),

$$k_{B_1} = 1 - 2 \cdot \xi \cdot \frac{\delta^*}{r_o} \tag{8}$$

Hence to account for the viscous blockage, k_{B_1} must be assigned a value or calculated from data using Eq. (8), and then, using Eq. (4)

and Eq. (7),

$$\Phi_o = \frac{1}{k_{B_1}} \cdot \frac{\overline{W}m}{\xi} \tag{9}$$

is the value of the flow function at the tip in terms of the measured flow rate. (The boundary layer on the hub is very thin and is ignored here).

4. CALCULATION OF THE FLOW RATE FOR CONDITIONS GIVEN AT THE TIP

The above Eq. (9) relates the flow function at the tip to the measured flow rate. It is desirable to relate the measured flow rate at a given rotor speed to the flow angle or incidence at the tip. The "design point" is taken as the condition at which the flow is at the design incidence angle at the tip ($\beta_{10} = \beta_{10}^*$) and the "design" flow rate is therefore a function of the rotor speed.

If β_{10} is given, using the notation of Fig. 1,

$$U_o = \frac{\pi N}{360} \cdot r_o \tag{10}$$

and

$$V_o = \frac{U_o}{\tan\beta_{10}} \tag{11}$$

If $\theta = T_t/T_{ref}$, where "ref" denotes a reference temperature (taken as $T_{ref} = 517.8°R$), then

$$X_o = \frac{V_o}{V_t} = \frac{1}{V_{ref}} \cdot \frac{\pi r_o}{260 \tan\beta_{10}} \left[\frac{N}{\sqrt{\theta}}\right] \tag{12}$$

where Eq. (9) and Eq. (10) have been used,

$$V_{ref} = \sqrt{2CpT_{ref}} \tag{13}$$

and

$$V_t = \sqrt{2CpT_t} \tag{14}$$

as described in Appendix A.

The total flow function at the tip is given by

$$\Phi_o = X_o(1 - X_o^2)^{1/\gamma-1} \qquad (15)$$

and using Eq. (9),

$$\overline{W}_m = k_{Bl} \cdot \xi \cdot \Phi_o \qquad (16)$$

From the definition of $\overline{W}_m$ in Eq. (2),

$$\dot{w}_m = \rho_t V_t \frac{\pi r_o^2}{144} \cdot \overline{W}_m$$

Using Eq. (16), Eq. (15) and introducing reference pressure and temperature,

$$\left[\frac{\dot{w}_m\sqrt{\theta}}{\delta}\right] = \rho_{ref} V_{ref} \cdot \left(\frac{\pi r_o^2}{144}\right) \cdot k_{Bl} \cdot \xi \cdot \left[X_o(1-X_o^2)\right]^{1/\gamma-1} \qquad (17)$$

In this equation $\rho_{ref} = \rho_t(\theta/\delta)$, where $\delta = (p_t/p_{ref})$, p_{ref} is a reference pressure (taken as p_{ref} = 29.92" Hg) and a perfect gas is assumed.

Since Eq. (12) relates the outer velocity X_o to the referred speed, $N/\sqrt{\theta}$ and the flow angle β_{1o}, Eq. (17) gives the referred flow rate in terms of these two variables for a given blockage factor. The HP9830A was programmed to graph the relationship given in Eq. (17). The result, for the blockage factor (0.99) measured in preliminary tests of the TRANSX compressor is given in Fig. 4. (The program is logged as HP-Sf001-15).

It should be observed that the design flow angle at the tip for the TRANSX compressor is $\beta_1 = 65°$. The design speed of the compressor is

30,460 rpm. In Fig. 4 choking of the inlet flow is seen to occur at about $\beta_1 = 57^{\circ}$ at this speed.

5. CALCULATION OF RADIAL PROFILES FOR A GIVEN FLOW RATE

For a given measured flow rate and stagnation conditions, $\overline{W}$ can be evaluated from the definition in Eq. (2). Φ_o is then given by Eq. (9).

At any value of R, Φ/Φ_o is given by Eq. (5) using the method of
iteration given in Appendix C, so that Φ is known. The non-dimensional 1
velocity X is obtained by solving the flow function as described in Appendix A. Then the non-dimensional local peripheral speed is

$$X_u = \frac{U}{V_t} = \frac{\pi}{360}\left(\frac{N}{\sqrt{\theta}}\right)\frac{r_o R}{V_{ref}} \tag{18}$$

and relative speed is

$$X_w = \frac{W}{V_t} = \sqrt{X^2 + Xu^2} \tag{19}$$

The local relative Mach number is then given by

$$M_w = \frac{W}{\sqrt{\gamma R_G T}} = \frac{W}{V_t}\,\frac{1}{\sqrt{\left(\frac{\gamma-1}{2}\right)(1-X^2)}} \tag{20}$$

and the flow angle by

$$\beta = \tan^{-1}\left(\frac{Xu}{X}\right) \tag{21}$$

This procedure can be used to obtain profiles of relative Mach numbers and flow angle for measured test data.

Alternatively, by specifying the flow angle at the tip Φ_o can be obtained from Eq. (12) and Eq. (15) and the radial profiles calculated in the same way.

The latter method was programmed for the HP9830A Calculator to obtain distributions of relative Mach number at "design" point of the TRANSX compressor as a function of the referred speed. (The program is logged as HP-Sf001-16). The results are shown in Fig. 5. It can be seen in Fig. 5 that at the design speed of 30,460 rpm, only a small region of the flow near the hub has a subsonic relative Mach number.

REFERENCES

1. Shreeve, R.P., "Calculation of Compressible Irrotational Streamlines in an Annulus" (unpublished) 1972.

2. Shreeve, R. P., "Calibration of Flow Nozzles Using Traversing Pitot-Static Probes", Naval Postgraduate School Technical Report NPS-57Sf73071A, July 1973.

3. Ames Research Staff, "Equations, Tables and Charts for Compressible Flow", NACA Report 1135, 1953.

Appendix A. Velocity from the Total Flow Function

The "total flow function" is the mass flux divided by the "limiting" or "stagnation" mass flux. A discussion of the flow function is given in Appendix A of reference 2. In addition to defining stagnation values of pressure, temperature and density, a "stagnation" or "limiting" velocity is introduced from the energy equation. Thus if

$$h_t = h + \frac{V^2}{2} \qquad \text{A(1)}$$

defines the total enthalpy,

$$V_t = \sqrt{2h_t} \qquad \text{A(2)}$$

is the "limiting" velocity obtained in an adiabatic expansion to zero temperature.

For a perfect gas, the equations relating static and stagnation properties become very simple if the velocity is made dimensionless by dividing by V_t[3]. Writing

$$X = V/V_t \qquad \text{A(3)}$$

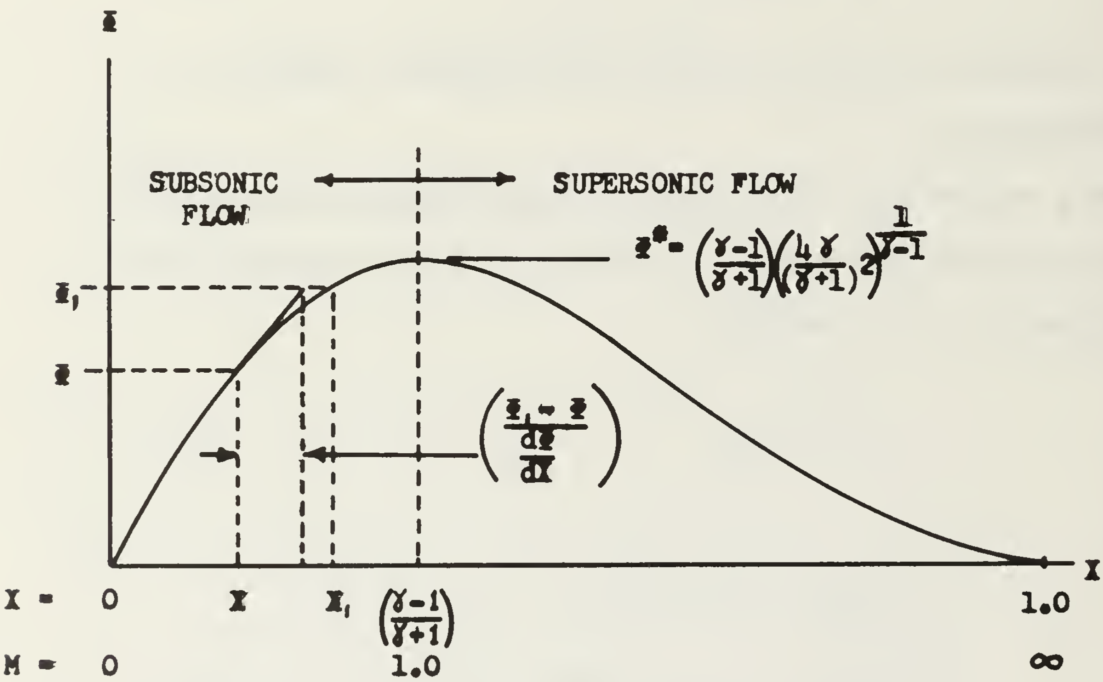
then

$$\left.\begin{aligned} T/T_t &= 1 - X^2 \\ p/p_t &= (1-X^2)^{\gamma/(\gamma-1)} \\ \rho/\rho_t &= (1-X^2)^{1/(\gamma-1)} \end{aligned}\right\} \qquad \text{A(4)}$$

and the total flow function, Φ, is then given by

$$\Phi \;=\; \rho V/\rho_t V_t \;=\; X(1 - X^2)^{1/(\gamma-1)} \qquad A(5)$$

In analysing internal compressible flows, the flow function in this form can often be used to eliminate density from the equations. In many cases, the flow rate and the stagnation properties are known and Equation A(5) must be solved to obtain the velocity. Care must then be taken to recognize the subsonic and supersonic roots as shown in the following sketch of Eq. A(5):



Differentiating Eq. A(5),

$$d\Phi/dX \;=\; \Phi[1/X - 2X/(\gamma - 1)(1 - X^2)] \qquad A(6)$$

For the <u>subsonic root</u> the procedure is as follows:

(i) guess $X < (\gamma-1/\gamma+1)$; $X = \Phi(1+\Phi)$ is suitable

(ii) calculate $\Phi = X(1 - X^2)^{1/(\gamma-1)}$

(iii) test for convergence; $|\Phi_1 - \Phi| < \varepsilon$? where Φ_1 is the given value

(iv) calculate $X = X + (\Phi_1 - \Phi)/(d\Phi/dX)$ using Eq. A(6)

(v) repeat from (ii) for convergence at (iii)

For the <u>supersonic root</u> choose an initial guess at (i) such that $X > (\gamma - 1)/(\gamma + 1)$ and use the above procedure without change.

Appendix B. Curve Fitting with $\sigma = a\varepsilon^n + m\varepsilon$

Compressor blading calculations involve the selection of a number of stream surfaces between which equal fractions of the total flow rate pass. By referring properties to conditions at the outer radius, distributions of quantities of order unity or less must be represented. A function which has been found to be useful to approximate smooth distributions calculated only at a limited number of points is the following;

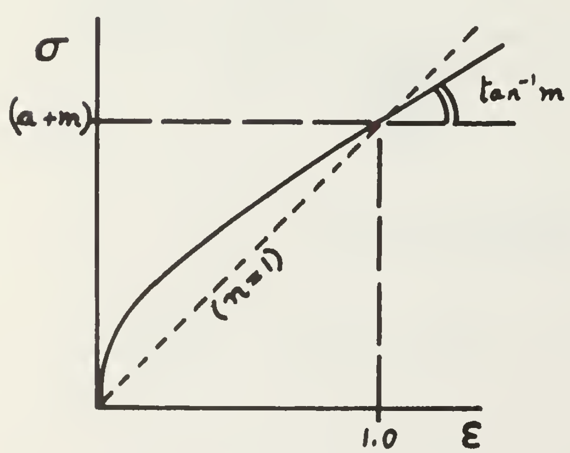
$$\sigma = a\varepsilon^n + m\varepsilon \qquad B(1)$$

The shape of $\sigma = \sigma(\varepsilon)$ given by Eq. B(1) depends on the range of ε and particularly on the value of n. Differentiating,

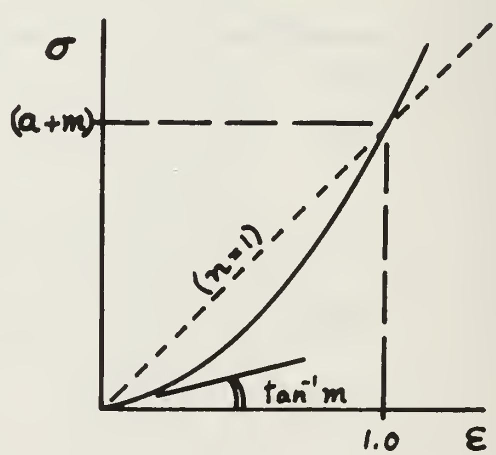
$$d\sigma/d\varepsilon = an\varepsilon^{n-1} + m \qquad B(2)$$

from which it is evident that there is a discontinuity in behavior at n = 1 for n going from values less than to greater than unity. This can be seen in the following sketches:

Case 1: $n < 1$



Case 2: $n > 1$



In Case 1, the slope is infinite at $\epsilon = 0$ and is given by the value of m as ϵ becomes large. In Case 2, the slope is given by m at $\epsilon = 0$ and increases as ϵ becomes large.

The function can be fitted to given data in a number of ways. There follows the analysis for fitting three arbitrary points exactly. (In application it is found that particular choices of data points can be made to improve the overall fit.)

If subscripts 1, 2 and 3 denote three given data points, the equations

$$\begin{aligned}\sigma_1 &= a\epsilon_1^n + m\epsilon_1\\ \sigma_2 &= a\epsilon_2^n + m\epsilon_2\\ \sigma_3 &= a\epsilon_3^n + m\epsilon_3\end{aligned} \qquad B(3)$$

must be solved to give the values of a, m and n.

Eliminating m between pairs of Eq. B(3) and 'a' from the resulting pair of equations, n is given by

$$\frac{\epsilon_1^{n-1} - \epsilon_2^{n-1}}{\epsilon_2^{n-1} - \epsilon_3^{n-1}} = \frac{\epsilon_3(\epsilon_2\sigma_1 - \epsilon_1\sigma_2)}{\epsilon_1(\epsilon_3\sigma_2 - \epsilon_2\sigma_3)} \qquad B(4)$$

When n is known, a and m are given by

$$a = \frac{\epsilon_2\sigma_1 - \epsilon_1\sigma_2}{\epsilon_2\epsilon_1^n - \epsilon_1\epsilon_2^n} \qquad B(5)$$

and

$$m = (\sigma_1 - a\epsilon_1{}^n)/\epsilon_1 = (\sigma_2 - a\epsilon_2{}^n)/\epsilon_2 \qquad B(6)$$

Thus it is necessary first to solve Eq. B(4). Writing

$$L = [\text{LHS of Eq. A(4)}] = [(\epsilon_1/\epsilon_2)^{n-1} - 1]/[1 - (\epsilon_3/\epsilon_2)^{n-1}]$$

or

$$L = (A^\eta - 1)/(1 - B^\eta) \qquad B(7)$$

where $A = \epsilon_1/\epsilon_2$, $B = \epsilon_3/\epsilon_2$ and $\eta = n - 1$.

Choose ϵ_1, ϵ_2 and ϵ_3 such that $0 < \epsilon_1 < \epsilon_2 < \epsilon_3$, then $A < 1$, $B > 1$ and η ($\gtrless 0$) is to be determined.

Differentiation of Eq. B(7) gives

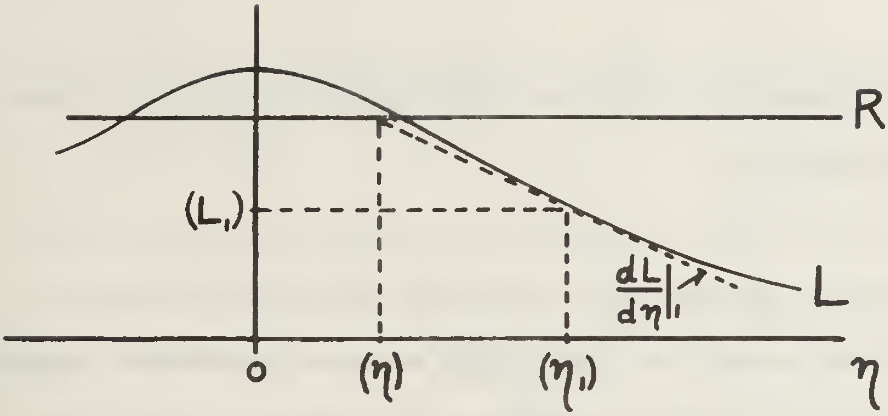
$$dL/dn = dL/d\eta = (A^\eta - B^\eta)/(1 - B^\eta)^2 \qquad B(8)$$

From an inspection of Eq. B(7) and Eq. B(8);

(i) If $\eta > 0$ $(n > 1)$, $L > 0$ and $(dL/d\eta) < 0$

(ii) If $\eta < 0$ $(n < 1)$, $L > 0$ and $(dL/d\eta) > 0$

Note that as $\eta \to \infty$, $L \to 0$. Hence the situation for obtaining a solution to Eq. B(4) is as shown in the sketch, where



$$R = \lceil \text{RHS of Eq. B(4)} \rceil = \frac{\varepsilon_3 (\varepsilon_2\sigma_1 - \varepsilon_1\sigma_2)}{\varepsilon_1 (\varepsilon_3\sigma_2 - \varepsilon_2\sigma_3)} \qquad B(9)$$

If an initial value of η is chosen ($= \eta_1$), then an improved guess is given by

$$\eta = \eta_1 + j \left[\frac{R - L_1}{\frac{dL}{d\eta}\Big|_1} \right] \qquad B(10)$$

where $j = \pm 1$ when $\eta_1 = \begin{smallmatrix} +ve \\ -ve \end{smallmatrix}$.

The initial guess should be made from an inspection of the curvature required to fit the data.

If we define the difference in slopes as follows:

$$C = \frac{\sigma_3 - \sigma_2}{\varepsilon_3 - \varepsilon_2} - \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} \qquad B(11)$$

then

$C > 0$ requires $\eta > 1$ $(\eta > 0)$

$C < 0$ requires $\eta < 1$ $(\eta < 0)$

Initial guesses of $\eta = -0.5$ for $C < 0$ and $\eta = 2.0$ for $C > 0$ have been found to be satisfactory

This procedure was applied to represent the distributions of total flow function and flow inclination angle at the TRANSX compressor rotor face. Data for eight streamlines calculated for equal mass flux increments were obtained from IBM 360 program IPP[1]. The HP9830A calculator was programmed to accept eight pairs of data values, to calculate a, m and n by curve fitting three specified pairs, to plot the resulting function over the range of interest and finally to plot the input data. The results are shown in Fig. 2 and Fig. 3 (for which the programs are logged as HP-Sf002-1 and HP-Sf001-3 respectively). It can be seen that the given data is very closely represented by the assumed expression, however the flow function is not represented explicitly in terms of R and must be obtained by iteration. The procedure is as follows:

(i) To obtain an initial guess for $\bar{\Phi}$ that is not greater than unity a power law is assumed

$$\bar{\Phi} = 1 - 0.03\,(1 - R_1^2)^2$$

(ii) Calculate

$$R^2 = 1 - ax^n - mx \text{ , where } x = 1 - \bar{\Phi} \text{ and}$$

$$\bar{\Phi} = \Phi/\Phi_o$$

(iii) Test for convergence; $\left| R_1^{\ 2} - R^2 \right| < \varepsilon$? where R_1 is the given value.

(iv) Calculate the slope at R^2; $dR^2/dx = -anx^{n-1} + m$

(v) Calculate $\Phi = \Phi - \left[\dfrac{R_1^{\ 2} - R^2}{\dfrac{dR^2}{dx}} \right]$

(vi) Repeat from (ii) for convergence at (iii).

Appendix C. Blockage Factor from Impact Pressure Measurements

Radial surveys of impact pressure at station 1 of the TRANSX compressor showed that the impact pressure was constant at radial locations more than 1" from the outer wall and the dynamic pressure was within 4% of the core value to within 0.2" from the wall. Treating the region near the wall as a boundary layer, the displacement thickness is given by

$$\delta^* = \int_0^{y_\infty} \left(1 - \frac{\rho u}{\rho_\infty \rho_\infty}\right) dy \qquad C(1)$$

where ∞ denotes the value at $r < 4.5"$ and y is measured radially inwards from the wall. In terms of the total flow function, Eq. C(1) can be written

$$\delta^* = \int_0^{y_\infty} \left(1 - \frac{\Phi}{\Phi_\infty}\right) dy \qquad C(2)$$

where, using Eq. A(4) and Eq. A(5)

$$\Phi = (p/p_t)^{1/\gamma} \sqrt{1 - (p/p_t)^{(\gamma-1)/\gamma}} \qquad C(3)$$

In terms of the difference Δ (ins. water col.) = $p_t - p$,

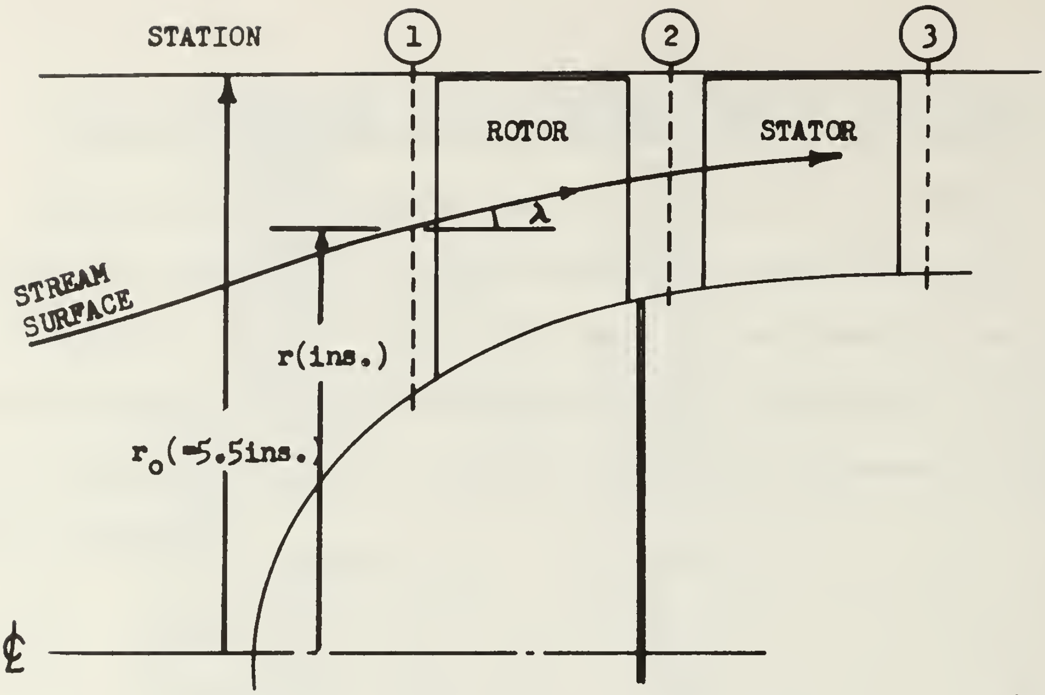
$$p/p_t = \left[\frac{1}{\Delta/p + 1}\right] \qquad C(4)$$

Equations C(2), C(3) and C(4) were programmed for the HP9830A and the following table gives the results for TRANSX compressor Run 8:

y	Δ (ins. water)	$(1 - \Phi/\Phi_\infty)$
~ 0	4.19	.2508818
.05	5.66	.1326629
.1	6.49	.0732442
.2	7.27	.0211151
.3	7.46	8.89409×10^{-3}
.5	7.57	1.89811×10^{-3}
.6	7.59	6.32237×10^{-4}
.7	7.58	1.26494×10^{-3}
.8	7.58	1.26494×10^{-3}
.9	7.60	0
1.0	7.60	0

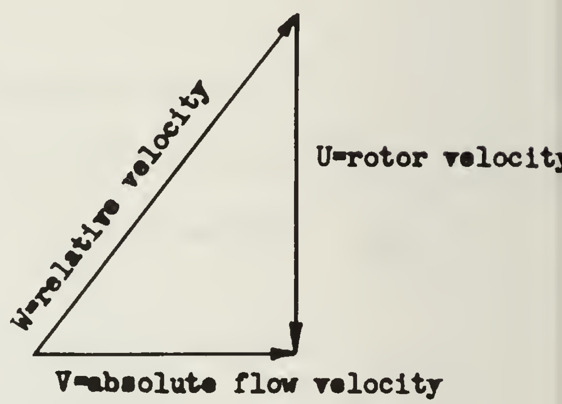
($\Phi_\infty = .07305442$ for $y_\infty = 1.0"$)

The integral in Eq. C(2) was calculated from the tabulated data using quadratic integration described in Appendix B of reference 2. For this data $\delta^* = 0.0210594$ and using Eq. 8 with $\xi = .70836$, the blockage factor $k_{B1} = 1 - 2.82344 \dfrac{\delta^*}{r_o} = 0.9892$.



NOTATION

M_w	relative Mach number
N	rotor RPM
p	pressure
R	r/r_o
R_G	gas constant
T	temperature(^oR)
$\dot{w}$	flow rate(lbs/sec)
$\overline{W}$	$\dot{w}/\rho_t V_t \pi r_o^2$
δ	p_t/p_{ref}
γ	ratio of specific heats
ρ	density
θ	T_t/T_{ref}



VELOCITY DIAGRAM AT STATION 1
IN THE STREAM SURFACE

SUBSCRIPTS

1	at station 1
i	inner wall
m	measured
o	outer wall
t	stagnation value
ref	at reference conditions

Figure 1. GEOMETRY AND PRINCIPAL NOTATION

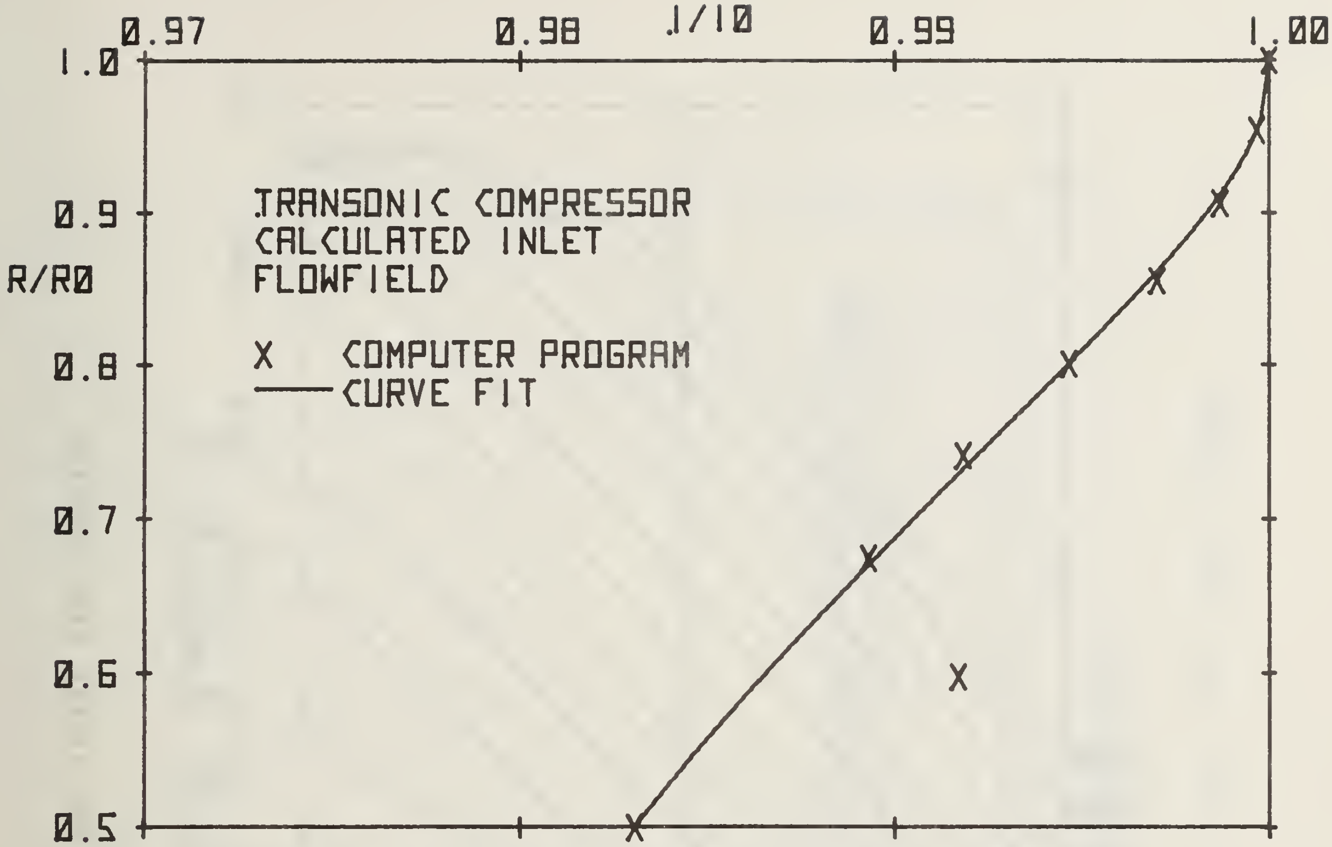


Figure 2. RADIAL DISTRIBUTION OF THE TOTAL FLOW FUNCTION

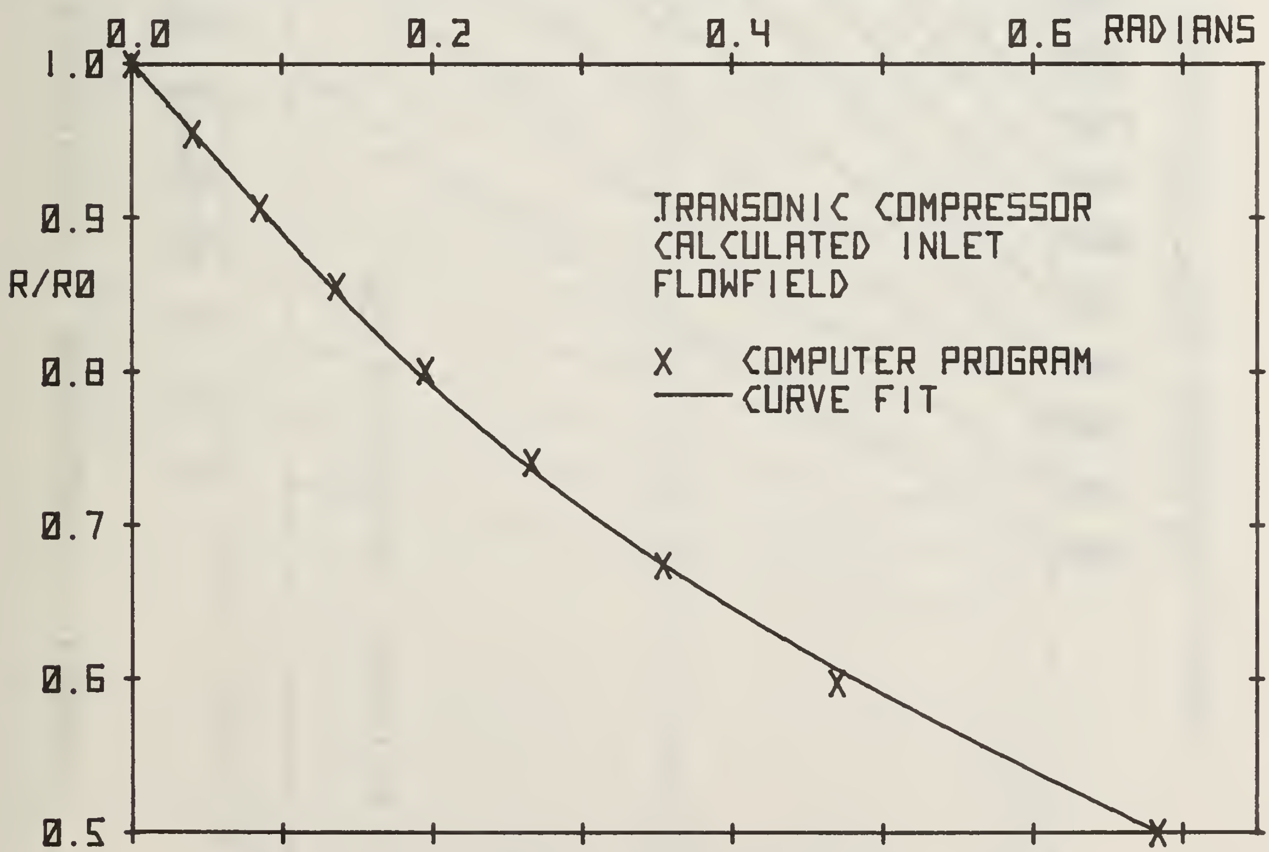


Figure 3. RADIAL DISTRIBUTION OF THE FLOW ANGLE

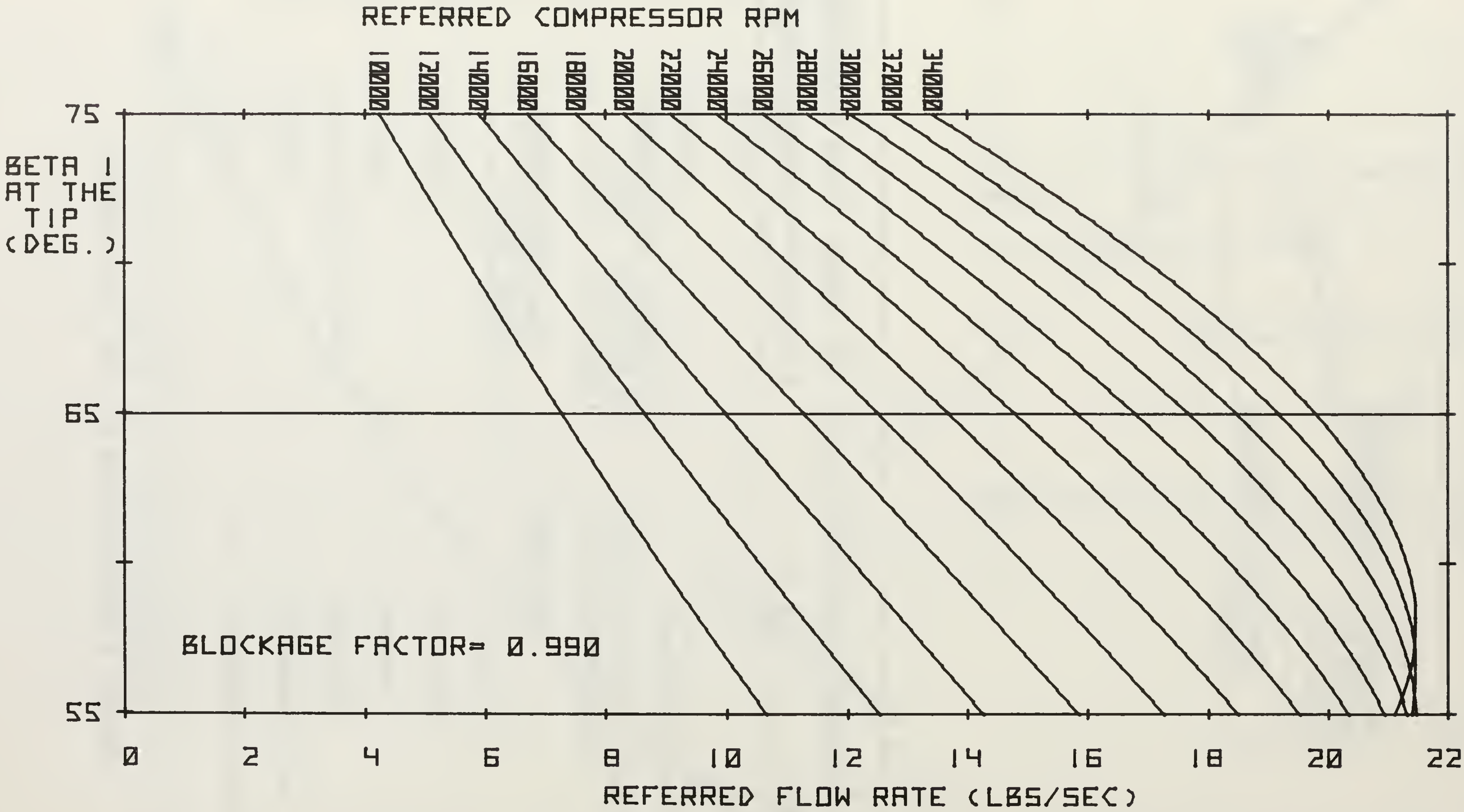


Figure 4. TRANSONIC COMPRESSOR - CALCULATED FLOW INTO THE ROTOR

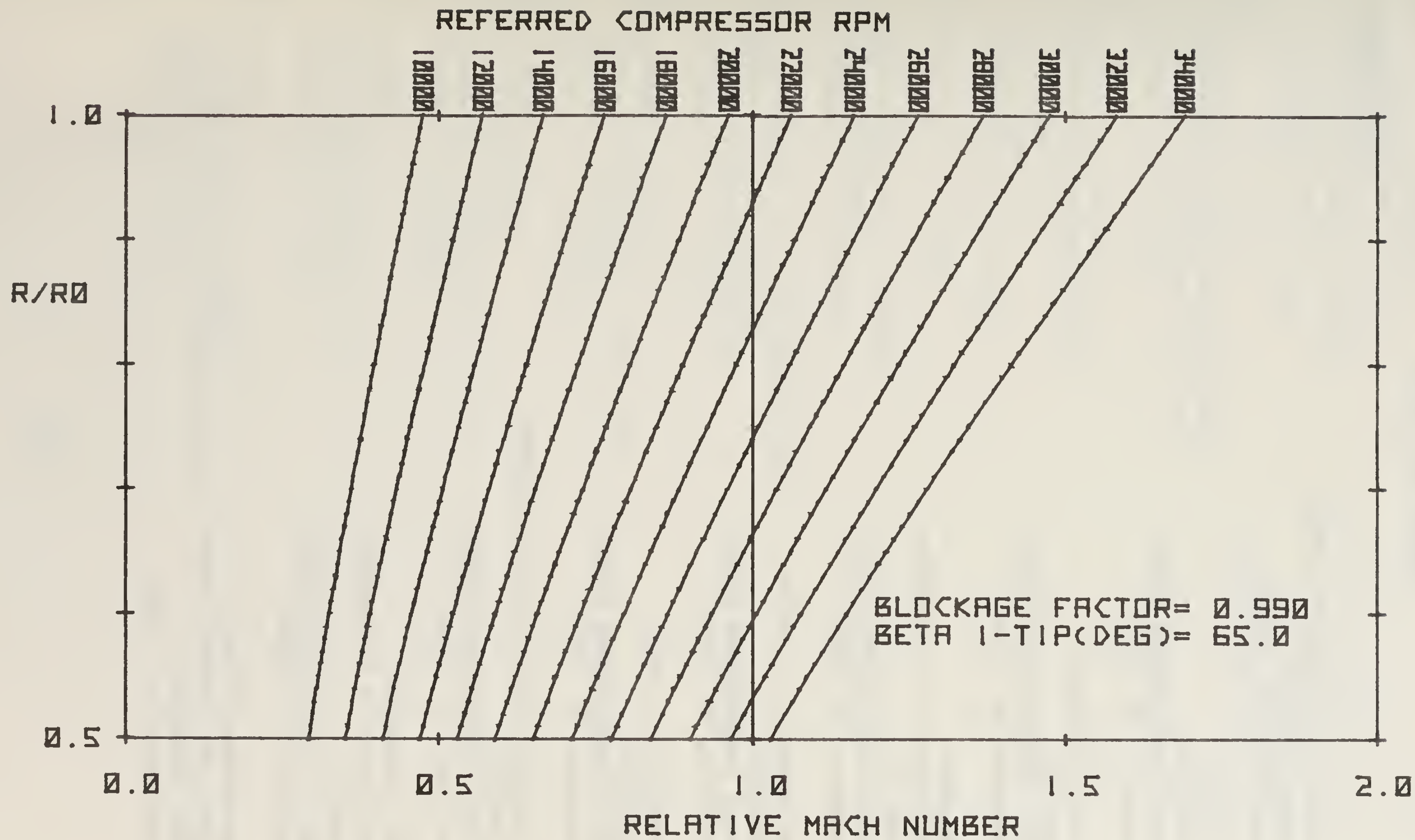


Figure 5. TRANSONIC COMPRESSOR - CALCULATED FLOW INTO THE ROTOR

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